

IOP Concise Physics
Atoms and Photons and Quanta, Oh My!
Ask the physicist about atomic, nuclear, and quantum physics
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Appendix G

Detecting spin

Suppose that you have a tiny bar magnet and put it in a uniform magnetic field; what will happen? As you can see in figure G.1, the north pole will be attracted to the south pole of the external magnet and the south pole will be attracted to the north pole with a force of equal magnitude but opposite direction. Therefore, the net force on the bar magnet is zero although there is a net torque tending to line the magnet up with the uniform field. So, if the bar magnet were shot through the field it would go through undeflected. Now, suppose that the field were nonuniform with the field being stronger near the south pole of the external magnet than near the north pole. This could be achieved by shaping the pole pieces of the external magnet as shown in figure G.2. If the bar magnet happened to have its south pole oriented upwards as in the figure, there would be a net force on the bar magnet which was down as shown. If the north pole were oriented upwards, there would be a net force upwards. If the bar magnet were oriented horizontally, there would be no net force on it. So, if a bar magnet were shot through this field (imagine you're shooting it straight down into the page in figure G.3), its straight-line path would be deflected depending on its initial orientation. A collection of randomly oriented little bar magnets would result in a vertical smear of emerging trajectories.

An experiment was performed by the German physicists Otto Stern and Walther Gerlach in 1922 which attempted to observe the effect. A small current loop has a magnetic field just the same as a small bar magnet. Therefore, with the Bohr model of orbiting electrons, the picture of the hydrogen atom in 1922, should behave like a tiny bar magnet and be deflected depending on its orientation when it entered the magnet. So it was expected that one would see a whole continuum of vertical deflections from the north-up to the north-down orientations. Instead what was seen was only two deflections, one up and one down. In terms of the Bohr model, this was not understandable.

Was it understandable in terms of the Schrödinger equation solutions of the hydrogen model? At first blush, the answer seems to be yes because the quantum number m specifies the allowed orientations of the angular momentum, so the orbit could not take on any orientation relative to the vertical axis, only those corresponding to allowed m values. But, the allowed values of m run from $-\ell$ to ℓ and there would always be an odd number of them because ℓ is always an integer. For example, for $\ell = 2$, $m_\ell = -2, -1, 0, 1, 2$, a total of five. So, the experiment is a puzzle. The results indicated that there were two values of m and so this would imply

that $m = -\frac{1}{2}, +\frac{1}{2}$ which would imply that $\ell = \frac{1}{2}$. But we know this cannot be true. This is where it is realized that the electron must have intrinsic angular momentum and it must have a quantum number $s = \frac{1}{2}$. Its magnetic quantum number should be labeled m_s to avoid confusion with the magnetic quantum m_ℓ number associated with orbital angular momentum. The possible m_s states, $\pm\frac{1}{2}$, are often referred to as 'spin up' and 'spin down'. There is a wonderful animation of the Stern–Gerlach experiment on Wikipedia. Finally, it should be noted that what is really being observed is the magnetic dipole moment of the electron, not the spin. A small magnet with N and S poles is called a magnetic dipole; the moment is a measure of the strength of the magnet. The fact that the deflections are the way they are lets us infer that the spin angular momentum quantum number must be and that there is an angular momentum associated with the magnetic moment.

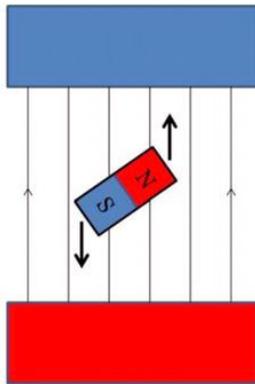


Figure G.1. Dipole in a uniform field.

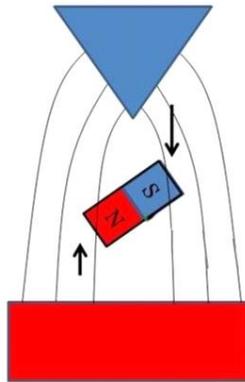


Figure G.2. Dipole in a non-uniform field.

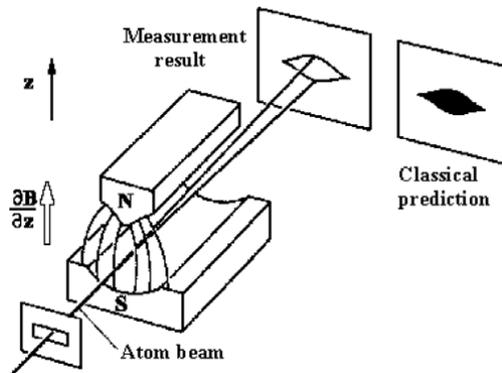


Figure G.3. The Stern-Gerlach experiment.