Kinetic energy is

$$K = E - mc^2 = \sqrt{p^2c^2 + m^2c^4} - mc^2 = mc^2\left(\sqrt{1+\frac{p^2c^2}{m^2c^4}} - 1\right).$$

Linear momentum is

$$p = mv/\sqrt{1-\beta^2} = m\beta c/\sqrt{1-\beta^2}$$

where

$$\beta = v/c\,.$$

Simple algebra leads to

$$K = mc^2\left(\left(1-\beta^2\right)^{-\frac{1}{2}} - 1\right).$$

You now do a binomial expansion

$$\left(1-\beta^2\right)^{-\frac{1}{2}} \approx 1 + \tfrac{1}{1!}\left(-\tfrac{1}{2}\right)\left(-\beta^2\right)^1 + \tfrac{1}{2!}\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)\left(-\beta^2\right)^2 + \cdots = \tfrac{1}{2}\beta^2 + \tfrac{3}{8}\beta^4 + \cdots$$

So,

$$K \approx mc^2\left(\tfrac{1}{2}\beta^2 + \tfrac{3}{8}\beta^4 + \cdots\right) = \tfrac{1}{2}mc^2\beta^2 + \tfrac{3}{8}mc^2\beta^4 + \cdots$$

The first term you recognize as

$$K_{classical} = \tfrac{1}{2}mv^2$$

and the second term is the lowest order correction

$$\Delta K_{classical} = \tfrac{3}{8}mc^2\beta^4,$$

so

$$\frac{\Delta K_{classical}}{K_{classical}} = \tfrac{3}{4}\beta^2$$

So, finally, for a 1% error,

$$0.01 = \tfrac{3}{4}\beta^2 \Rightarrow \beta = 0.116$$

which is your $v/c<1/10$, I guess!